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Finite element modelling of composite structures under crushing load

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Abstract

This paper details the theory and implementation of a composite damage model, addressing damage within a ply (intralaminar) and delamination (interlaminar), developed for the simulation of the crushing of laminated composite structures. It includes a more accurate determination of the characteristic length to achieve mesh objectivity in capturing intralaminar damage consisting of matrix cracking and fibre failure, a load-history dependent material response, an isotropic hardening nonlinear matrix response, as well as a more physically-based interactive matrix damage mechanism. The developed damage model requires a set of material parameters obtained from a combination of standard and non-standard material characterisation tests. The fidelity of the model mitigates the need to manipulate, or “calibrate”, the input data to achieve good agreement with experimental results. This intralaminar damage model was implemented as a VUMAT subroutine, and used in conjunction with an existing interlaminar damage model, in Abaqus/Explicit. This approach was validated through the simulation of the crushing of a cross-ply composite tube with a tulip-shaped trigger, loaded in uniaxial compression. Despite the complexity of the chosen geometry, excellent correlation was achieved with experimental results.

1 Introduction

Over recent years, there has been a concerted effort by civil airframe manufacturers to develop lighter aerostructures with reduced operating costs. This has driven the increased use of carbon fibre reinforced polymer (CFRP) materials in the primary structure of the latest generation of passenger aircraft. As a result, there has been a considerable focus [1-4] on investigating the energy absorbing characteristics of CFRP to determine the crashworthiness of composite aerostructures.

Composite materials offer superior potential as energy absorbers due to their high specific strength and the multitude of possible different energy-dissipating damage modes of matrix cracking, fibre failure and delamination[5]. However, the effective design of these energy absorbers is a complex undertaking due to the difficulty in predicting the multiple concurrent failure modes and their interactions. This lack of robustness and predictive capability of current numerical modelling tools, has meant that the crashworthiness assessment of composite subcomponents currently relies heavily on physical testing which is both time consuming and expensive.

Energy absorbing structures with varying geometries have been extensively investigated in the literature. Typically, self-supporting geometries have been adopted for practical applications [6], including tube and tube-like structures [7-9], channel sections [10, 11], as well as corrugated webs [12, 13]. In particular, circular tubes have been studied extensively in the literature [14, 15] to determine the effects of composition [16], layup [17], and trigger mechanism [18, 19]. Analysis of loading on a typical section of material in a crushing structure is highly complex and three-dimensional. Much of the existing computational damage models available in

commercial finite element packages, which track the initiation and evolution of damage through a structure, have been developed using plane stress shell elements. These elements assume that through thickness stresses are negligible, which does not hold true for structures being crushed. One example is material model type 54 (Mat54), a shell-based formulation implemented in LS-DYNA[20] utilising an approach developed by Chang and Chang [21]. Mat54 was used by Ghasemnejad et al. [22] and Feraboli et al. [23] to capture closed and open sectioned specimen crushing respectively. To achieve good correlation with experimental data, the input parameters were determined by trial and error [23]. Similarly, the ply type 7 model, based on Ladeveze and Le Dantec's [24] work, implemented for PAM-CRASH was used by Joosten et al. [4] and Johnson and David [1] to simulate the response of hat-shaped and C-shaped channel sections under crushing loads. Similar to Mat54, some input parameters required substantial calibration against experimental test data in order for this material model to produce the desired response, rendering these models incapable of being a reliable predictive tool. Close examination of these results suggests that for damage-inducing loads, a 2D shell formulation is insufficient.

The complexity of composite crushing invariably results in highly localised and rapid load redistribution and consequently the damage process needs to take into account the loading history on the material. Much of the existing techniques do not properly account for the effect of loading/unloading and load reversal, particularly for the inelastic shear behaviour of the matrix material. Donadon et al. [25] and later Faggiani and Falzon [26] produced damage models which considered unloading and reloading behaviour. These two models were able to yield good predictions of the impact response for a simple composite plate and a stiffened composite panel,

respectively, where the damage was matrix-dominated. However, the differences in the anticipated loading conditions between an impact and a crush event have identified limitations in these models. Puck and Schürmann [27] showed that the assessment of matrix damage requires consideration of local interactions which was incorporated into the work of Shi et al [28] as well as many others. Raimondo et al. [29] used an energy-based, interactive approach that took into account the contribution of each loading direction to the overall energy balance within each element. However, the damage model did not account for loading and reloading which may have contributed to the over prediction of the force response during the impactor rebound phase after impact, while still in contact with the composite plate. Another issue with the model was the use of a crack saturation density parameter, which was obtained by trial and error, and hence is effectively a calibration parameter.

An accurate estimate of a characteristic length measure, associated with each finite element, is required to achieve mesh independence for a continuum damage mechanics (CDM) based smeared crack finite element (FE) damage model. Bazant and Oh [30] pioneered the concept of a crack band to prevent damage localisation, leading to zero energy dissipation, and preserve mesh objectivity in the FEA of softening material. Subsequent work by Bazant and Cedolin [31] details localisation issues for different classes of constitutive laws, including CDM. Jirasek and Bazant [32] demonstrated the use for a characteristic length to scale the softening behaviour. The effectiveness of this approach has been analytically shown by Oliver [33]. Much of the existing composite damage models use a coarse estimate of this quantity. The Abaqus in-built composite damage model, based on the work by Matzenmiller et al.

[34], approximates the characteristic length as the cube root of the elemental volume, which is increasingly inaccurate as the aspect ratio increases. Donadon et al's [25] model resolved this issue by calculating the characteristic length directly from the shape functions of the element, but with restricted fracture plane orientations. Matrix failure in a non-zero degree ply would generate a fracture plane that cannot be represented using this method. A more general algorithm is required to determine the characteristic length for the full spectrum of possible orientations of the fracture plane.

INSERT Table 1: Summary of comparison between existing models and the present model.

This paper presents a finite-element based damage model, formulated for 3D solid elements and tailored for virtual crush testing, which is able to capture the full suite of damage mechanisms and their interaction, within a continuous fibre laminated composite structure. The proposed model presents numerous advances over current approaches. Load history effects were incorporated into the interaction of damage. In particular, the material nonlinearity was accounted for during unloading and load reversals. A unified matrix damage mechanism was developed to include energy contribution for multi-axial loading which includes material nonlinearity. A robust characteristic length algorithm for arbitrarily oriented fracture plane was developed and incorporated. This damage model is combined with established interlaminar damage as well as friction models to form a complete package for the simulation of damage in composite structures. The theory and implementation of the intralaminar damage model is presented in Section 2. The numerical results are compared with

experimental test data in Section 4, demonstrating the predictive capability of the developed model.

2 Intralaminar damage model – theory and implementation

2.1 Theoretical foundations – quantifying damage effects in the material

The intralaminar damage model is based on continuum damage mechanics (CDM) for 3D stress states. CDM was first proposed by Kachanov and was subsequently applied to micro-cracking in composite materials by Talreja [35], who proposed an energy-based constitutive relationship that includes the effect of damage for composite laminate plies through a homogenised damage field vector. Anisotropy in the laminate lead to the assignment of separate damage parameters for damage modes associated with each direction [36]. Developments by Chaboche [37] and Lemaitre [38], lead to a method to determine the behaviour of a material under damage-inducing loads. CDM assumes that before macroscopic fracture occurs, microscopic cracks and voids form within the material being loaded [38], causing a reduction in the effective load bearing area which, for the 1-D case, can be quantified by a damage parameter:

$$d = \frac{A - \tilde{A}}{A} \quad (1)$$

where A is the pristine load bearing area and \tilde{A} is the reduced load bearing area in the presence of microscopic damage. The damage parameter from different damage modes combine to form the damage matrix $[D]$ which relates the stress vector in the damaged material, σ , to the effective stress vector, $\tilde{\sigma}$, which would have been experienced by the material, had damage not occurred.

$$\sigma = [D]\tilde{\sigma} \quad (2)$$

Operating on the assumption of strain equivalence, the degraded stiffness matrix, $[\tilde{C}]$, of the damaged material may be expressed as a function of the pristine material stiffness $[C]$,

$$[\tilde{C}] = [D][C] \quad (3)$$

which leads to a softening of the material once damage has initiated. As more energy is dissipated via damage formation in the material, the load-bearing capacity of the material reduces. The transmitted stress reduces to zero when the volumetric strain energy dissipated, g , reaches a critical value, signifying the complete failure of the material. This point is defined by the failure strain, $\varepsilon_{failure}$:

$$\int_0^{\varepsilon_{failure}} \sigma d\varepsilon = g \quad (4)$$

Damage must be irreversible to ensure that its formation is thermodynamically consistent. Hence any damage parameter must be monotonically increasing. Damage mechanisms present within a unidirectional continuous fibre ply are captured by three damage parameters; two to account for the tension and compression in fibre-dominated damage and one for matrix-dominated damage. This is discussed in more detail in Section 2.2.3. To capture the whole structural response to damage inducing loads on laminated composite structures, an existing cohesive damage model is utilised in conjunction with the developed intralaminar model.

2.2 Detailed theory and implementation of the damage model

The developed intralaminar damage model assesses damage in the two phases of a continuous fibre unidirectional composite ply (Fig. 1). Fibre-dominated damage is primarily associated with loading along the fibre direction. The anticipated damage will occur in the form of net fibre breakage in tension and predominantly fibre kink-

band formation when loaded in compression. Matrix-dominated damage is primarily associated with transverse and shear loading which leads to plasticity and formation of cracks in the matrix material. The use of this 3D FE method necessitates the determination of a characteristic length to correctly scale the critical energy density.

INSERT Figure 1: Flowchart showing the processes within the intralaminar damage model.

2.2.1 Characteristic length calculation

2.2.1.1 The role of the characteristic length in achieving mesh objectivity

When the strain energy density within an element exceeds g , it no longer transmits loading and is deemed to have failed. g is intrinsically linked to the critical energy release rate, G , the energy required to create a unit area of fracture surface, which can be measured experimentally. G and g are related by a characteristic length where different fracture modes have different energy release rates and corresponding characteristic lengths, l_{mode} . Through the characteristic length, g is scaled so that different mesh densities return the same total energy absorption at fracture.

Oliver [33] showed that by estimating the crack size within the element, a proper characteristic length (l_{mode}) can be deduced to scale the experimentally determined critical energy release rate (G_{mode}^{dir}) to the critical volumetric strain energy (g_{mode}^{dir}) for a given mode and direction:

$$g_{mode}^{dir} = \frac{G_{mode}^{dir}}{l_{mode}} \quad (5)$$

Oliver's analysis was performed on a 2D grid. An effective crack length was calculated according to how the crack partitions the element. From this, the characteristic length was calculated as the ratio between the area of the element and the effective crack length. This method was shown to be consistent with theoretical predictions for simple test cases. Generalising this concept to 3D yields:

$$l_{mode} = \frac{V}{A} \quad (6)$$

The elemental volume (V) can be obtained from the FE simulation. In calculating the crack area for an arbitrarily oriented crack surface (A), the orientation of the material coordinate system, with respect to the element, and the rotation of the fracture plane must be taken into consideration.

2.2.1.2 Implementation of characteristic length calculation

The fracture surface is defined by a unit normal vector (\hat{n}) in an arbitrary hexahedral element (Fig. 2). This normal vector contains information about the material coordinate system as well as the fracture plane rotation.

INSERT Figure 2: Definition of unit normal vector (\hat{n}) and points of edge intersection (p_k) for an arbitrary fracture plane (shaded) within a hexahedral element.

The algorithm determines the points (p_k) where the fracture plane intersects with the elemental boundary formed by connecting adjacent nodes. The triangular areas (A_l) enclosed by adjacent intersection points (p_k) and the centre are then determined. Summing these, the total fracture plane area is approximated. This calculation is completed for each element in the model.

This procedure requires material coordinate system information as well as initial nodal coordinates of the elements, which are not provided by the VUMAT input/output interface. This is resolved by reading the input file itself in the pre-processing stage to extract this information. All elements in the model are assigned internal element numbers during the simulation process. An internal Abaqus utility routine (vgetinternal) was used to match the element data obtained from the input file to the correct element.

One of the major advantages of this method is that it is able to operate for models where the global, elemental and the material coordinate systems do not align, allowing greater freedom in how the structure is meshed. Additional flexibility comes from allowing each element to have an independent material coordinate system so that curved structures can be handled more accurately. It also enables the use of elements with a range of aspect ratios without significantly effecting accuracy.

Eight-node linear reduced integration solid elements, with one integration point at the centroid, are used in Abaqus/Explicit and consequently the fracture plane is assumed to pass through the centroid of each failed element. With further mesh refinement, an arbitrary macro-scale crack can be represented by a connected series of failed elements. It is also assumed that the element does not become concave. This is appropriate because built into the FE package [39] is a mechanism to prevent elements from becoming inverted.

2.2.2 Modelling fibre-dominated damage

Fibre-dominated damage represents the damage, which affects the longitudinal behaviour of a unidirectional prepreg (Fig. 3).

INSERT Figure 3: Fibre-dominated fracture with associated fracture plane.

In tension, this manifests as the breakage and pull out of fibres from the surrounding matrix. On the other hand, compressive damage causes the fibres to buckle and break during the formation of kink bands. The tensile and compressive characteristics are considerably different. Hence a separate damage parameter is defined for each mode. To determine the softening of the longitudinal modulus due to damage, the effects of tensile and compressive damage are combined.

2.2.2.1 Damage initiation

The point of damage initiation for the fibre-dominated mode is found by comparing the strain to the damage initiation strain (ε_{11}^{OT} and ε_{11}^{OC} for tension and compression respectively). An initiation function (F_{11}^T and F_{11}^C for tension and compression respectively) is defined for both tensile and compressive loading as follows:

$$F_{fib}^{T/C} = \left(\frac{\varepsilon_{11}}{\varepsilon_{11}^{OT/OC}} \right)^2 \quad (7)$$

where the initiation strains are determined from the longitudinal elastic modulus (E_{11}) and strengths in tension and compression (X^T and X^C respectively).

$$\varepsilon_{11}^{OT/OC} = \frac{X^{T/C}}{E_{11}} \quad (8)$$

When the initiation function for any damage mode reaches unity, the initiation criterion is met and the damage begins to propagate.

2.2.2.2 Damage evolution

A bilinear response is assumed for both tensile (Fig. 4) and compressive loading in the fibre direction. The bilinear model is appropriate for fibre dominated tension as it has brittle behaviour but it is an approximation for fibre dominated compression.

Other criteria, such as the fibre misalignment shear stress kinking model [40], give a more accurate damage initiation strength in compression. However, the initiation strength is not as important as the total energy consumption of the damage process, which is the basis for the proposed model. Hence the bilinear model is a compromise between simplicity and accuracy when used to represent fibre compression.

A positive linear stiffness describes the stress-strain behaviour prior to damage initiation. After initiation, the tangent modulus becomes negative due to the degradation of the elastic modulus by the damage parameter (d_{fib}) according to Eq. (3). The tensile fibre-dominated damage parameter (d_{11}^T) is found by comparing the current strain (ε_{11}) with the failure strain (ε_{11}^{FT}):

$$d_{11}^T(\varepsilon_{11}) = \frac{\varepsilon_{11}^{FT}}{\varepsilon_{11}^{FT} - \varepsilon_{11}^{OT}} \left(1 - \frac{\varepsilon_{11}^{OT}}{\varepsilon_{11}} \right) \quad (9)$$

The tensile failure strain (ε_{11}^{FT}) is determined by combining Eqs. (4) and (5), and is a function of the fibre-dominated tensile critical energy release rate (G_{fib}^T) and the corresponding characteristic length (l_{fib}).

$$\varepsilon_{11}^{FT} = \frac{2G_{fib}^T}{X^T l_{fib}} = \frac{2g_{fib}^T}{X^T} \quad (10)$$

The fibre tensile critical energy release rate (G_{fib}^T) is found experimentally, representing the energy consumed in creating an area of crack under uniaxial tensile

loading in the longitudinal direction. Scaling G_{fib}^T with the characteristic length yields the corresponding critical volumetric energy density, g_{fib}^T .

INSERT Figure 4: Bilinear stress-strain law (shaded area is the critical volumetric strain energy release rate g_{fib}^T) and associated damage parameter growth (bold dashed line).

The same approach is applied for compressive loading, which leads to:

$$d_{11}^C(\varepsilon_{11}) = \frac{\varepsilon_{11}^{FC}}{\varepsilon_{11}^{FC} - \varepsilon_{11}^{OC}} \left(1 - \frac{\varepsilon_{11}^{OC}}{\varepsilon_{11}} \right) \quad (11)$$

$$\varepsilon_{11}^{FC} = \frac{2G_{fib}^C}{X^C l_{fib}} = \frac{2g_{fib}^C}{X^C} \quad (12)$$

When unloading and load reversal is introduced, the damage caused by tensile and compressive loading will interact. It is assumed that the growth of damage in the tensile mode does not significantly affect the response in compression when the loading is reversed. Even though fibre breakage has occurred, it is assumed that the surrounding matrix material is still able to support the fibre when it experiences compressive loading, so the compressive modulus is maintained. This is illustrated in Fig. 5 where unloading along path 3 and reversing the load such that the material is now in compression will result in an initial elastic stiffness response represented by path 4. However, the reverse is not true. In compressive loading induced damage, fibre breakage occurs under kink band formation. Hence the stiffness is reduced when the material is subsequently loaded in tension. Therefore, the growth in the compressive damage parameter will also cause the tensile damage parameter to grow and the stiffness to soften (paths 6 to 7).

INSERT Figure 5: Stress-strain response during fibre direction loading/unloading: paths 1-2, tensile loading; 3, unloading; 4-5, loading in compression; 6-8, compressive unloading and tensile reloading until failure.

To achieve the interaction shown in Fig. 5, the modulus is reduced according to the longitudinal damage parameter defined as

$$d_{fib} = \begin{cases} d_{11}^c & \varepsilon_{11} < 0 \\ d_{11}^T & \varepsilon_{11} \geq 0 \text{ and } d_{11}^T > d_{11}^c \\ d_{11}^c & \varepsilon_{11} \geq 0 \text{ and } d_{11}^T < d_{11}^c \end{cases} \quad (13)$$

This fibre dominated damage interaction mechanism is similar to that employed by other authors [25, 26, 41, 42]. The effective stress vector ($\{\bar{\sigma}\}$) before the application of the softening effect of damage is determined by Hooke's law,

$$\{\bar{\sigma}\} = [C] \{\varepsilon\} \quad (14)$$

where $[C]$ is the stiffness matrix of the orthotropic laminate, which is determined by the elastic properties in the fibre (11), transverse (22) and thickness (33) directions.

$[C]$

$$= \begin{bmatrix} \frac{1 - \nu_{23}\nu_{32}}{E_{22}E_{33}\Delta} & \frac{\nu_{21} - \nu_{23}\nu_{31}}{E_{22}E_{33}\Delta} & \frac{\nu_{31} - \nu_{21}\nu_{32}}{E_{22}E_{33}\Delta} & 0 & 0 & 0 \\ \frac{\nu_{21} - \nu_{23}\nu_{31}}{E_{22}E_{33}\Delta} & \frac{1 - \nu_{13}\nu_{31}}{E_{11}E_{33}\Delta} & \frac{\nu_{32} - \nu_{12}\nu_{31}}{E_{11}E_{33}\Delta} & 0 & 0 & 0 \\ \frac{\nu_{31} - \nu_{21}\nu_{32}}{E_{22}E_{33}\Delta} & \frac{\nu_{32} - \nu_{12}\nu_{31}}{E_{11}E_{33}\Delta} & \frac{1 - \nu_{12}\nu_{21}}{E_{11}E_{22}\Delta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G_{13} \end{bmatrix} \quad (15)$$

$$\text{Where } \Delta = \frac{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13}}{E_{11}E_{22}E_{33}}$$

Once damage has initiated, the elastic moduli are degraded according to Eqs. (2) and (3). In order for the stiffness matrix, $[C]$, to remain positive-definite as damage progresses, the relationship in Eq. (16) must be maintained.

$$\frac{\nu_{ij,d}}{E_{ii,d}} = \frac{\nu_{ij}(1 - d_{ii})}{E_{ii}(1 - d_{ii})} = \frac{\nu_{ji}(1 - d_{jj})}{E_{jj}(1 - d_{jj})} = \frac{\nu_{ji,d}}{E_{jj,d}} \quad (16)$$

This approach is consistent with the experimentally observed Poisson's ratio degradation that accompanies the progression of damage in composite materials [43]. Applying the damage parameter in Eq. (13) to the longitudinal component of Eq. (14), the damaged stress in the longitudinal direction can be determined by:

$$\sigma_{11} = (1 - d_{fib})\bar{\sigma}_{11} \quad (17)$$

2.2.2.3 Implementation of fibre-dominated damage

To ensure that the bilinear law is preserved, a necessary condition is that the failure strain must be greater than the initiation strain. As a result, the characteristic length must satisfy

$$l_{fib} \leq \frac{2G_{fib}^{dir}}{X^{dir}\varepsilon_{fib}^{o,dir}} \quad \text{for } dir = T \text{ and } C \quad (18)$$

for all elements in the mesh. This criterion imposes an upper limit on the characteristic length, hence restricting the maximum size of elements in the model.

2.2.3 Modelling matrix-dominated damage

Matrix-dominated damage represents the damage sustained that primarily affects the transverse behaviour (Fig. 6).

INSERT Figure 6: Matrix-dominated fracture with associated fracture plane.

In uniaxial tension, the fracture plane forms perpendicular to the principal loading direction. However, for compressive and shear loading, the fracture occurs via shear cracking along a rotated fracture plane. Puck and Schürmann [27] developed a set of damage initiation criteria that is based on this fracture plane, as defined by a rotation of θ about an axis parallel to the fibre direction shown in Fig. 7.

INSERT Figure 7: Coordinate system attached to the fracture plane (1,N,T) [27] relative to the material coordinate system (1,2,3).

The transformation matrix outlined in Eq. (19) is used to convert between the fracture plane coordinate system (FPCS) and the material coordinate system (MCS) and is applied to both stress and strain tensors (Eqs. (20) and (21)).

$$[T(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} \quad (19)$$

$$\{\sigma_{FPCS}\} = [T(\theta_{fp})]\{\sigma_{MCS}\}[T(\theta_{fp})]^T \quad (20)$$

$$\{\varepsilon_{FPCS}\} = [T(\theta_{fp})]\{\varepsilon_{MCS}\}[T(\theta_{fp})]^T \quad (21)$$

2.2.3.1 Damage initiation

Puck and Schürmann [27] proposed that failure in the matrix phase is caused by stresses on this fracture plane, resulting in the matrix-dominated damage initiation functions in Eqs. (22) and (23).

$$F_{mat}^T = \left(\frac{\sigma_{NN}}{Y^T}\right)^2 + \left(\frac{\tau_{NT}}{S_{23}^A}\right)^2 + \left(\frac{\tau_{1N}}{S_{12}}\right)^2 \quad \text{for } \sigma_{NN} > 0 \quad (22)$$

$$F_{mat}^C = \left(\frac{\tau_{NT}}{S_{23}^A - \mu_{NT}\sigma_{NN}}\right)^2 + \left(\frac{\tau_{1N}}{S_{12} - \mu_{1N}\sigma_{NN}}\right)^2 \quad \text{for } \sigma_{NN} \leq 0 \quad (23)$$

The damage initiation functions compare the loading against the resistance on the fracture plane comprising of tensile and compressive transverse strengths (Y^T and Y^C), longitudinal and transverse shear (S_{12} and S_{23}^A), pseudo-friction coefficients (μ_{NT} and μ_{1N}) and the normal stress on the fracture plane (σ_{NN}). Strengths, as measured on a unidirectional laminate, were used in Eqs. (21) and (22). Camanho et al [44] have shown that in-situ shear strength is dependent on both ply configuration as well as the local ply thickness. Determining local ply thickness as well as local ply configuration would be very computationally expensive, particularly when element deletion is involved as the local conditions would have to be recalculated constantly. Furthermore, the action of damage can reduce the constraining effect of neighbouring plies and thus, the in-situ effects. Hence, the unidirectional values are used as a conservative representation of the true local strength.

The fracture plane angle (θ_{fp}) is not initially known for a general loading state. The stresses ($\sigma_{NN}, \tau_{NT}, \tau_{1N}$) in the initiation functions (Eqs. (22) and (23)) are a function of inclination angle of the fracture plane (θ). The initiation functions must first be maximised with respect to θ . Damage initiation occurs when the maximised initiation functions have a value greater than unity. Once the angle is determined, it will not change for this element for the remainder of the simulation as shear micro-cracking has occurred and any further fracture will preferentially occur on this plane. Brent's algorithm [45] was used to efficiently determine the orientation of the fracture plane. With reference to Fig. 8, when $\sigma_{NN} > 0$, the damage initiation profile is defined by a semi-ellipse that intersects the stress axes at the transverse tensile and shear strengths. When $\sigma_{NN} < 0$, the profile is defined by a line passing through point A and the shear

strength, where point A is derived from the observed fracture plane angle (θ_f) of 53° for uniaxial transverse compression [27].

INSERT Figure 8: Defining the damage initiation profile (dashed line) via the material properties on the friction plane, where $j = 1, T$, and inset represents the stress state at point A.

The transverse shear strength (S_{23}^A) is defined in terms of the transverse compressive strength [40], i.e.:

$$S_{23}^A = \frac{Y^c}{2 \tan(\theta_f)} \quad (24)$$

The slope of the linear section of the damage initiation profile in Fig. 8 can be interpreted as a friction coefficient that aids in resisting shear loading [40]:

$$\mu_{NT} = -\frac{1}{\tan(2\theta_f)} \quad (25)$$

The same analysis is repeated for the thickness direction response via:

$$\mu_{1N} = \frac{S_{12}}{S_{23}^A} \mu_{NT} \quad (26)$$

Combining both the transverse and thickness direction profiles using a quadratic relationship gives an overall damage initiation surface for matrix damage shown in Fig. 9.

INSERT Figure 9: Damage initiation surface in the stress space defined by the friction plane.

The shear model splits the overall strain into its elastic and inelastic components (Eq. (27)). A cubic function (Eq. (28)) was used to describe the nonlinear behaviour of composites under shear loading when damage has not yet initiated ($d_{mat} = 0$).

$$\gamma_{ij} = \gamma_{ij,el} + \gamma_{ij,in} \quad i \neq j \quad (27)$$

$$\tau(\gamma_{ij}) = c_1 \gamma_{ij}^3 - \text{sgn}(\gamma_{ij}) c_2 \gamma_{ij}^2 + c_3 \gamma_{ij} \quad (28)$$

With the inclusion of inelastic strain in the shear response, an isotropic hardening relationship was adopted to deal with unloading and load reversal. Fig. 10 shows an initial stress state (γ^t, τ^t) , which is reached after partial unloading along the secant shear modulus (G_{ij}) . The stress state after subsequent reloading to $\gamma^{t+\Delta t}$ depends on whether plastic yielding occurs. Initially, the stress is assumed to increase elastically to $\tau_E^{t+\Delta t}$. However, as $\tau_E^{t+\Delta t} > \tau(\gamma^t)$, yielding has occurred, which results in the increased inelastic strain of $\gamma_{in}^{t+\Delta t}$ and stress reduced to $\tau^{t+\Delta t}$.

INSERT Figure 10: Calculating shear stress from shear strain using the elastic predictor method progressing from old stress state τ_0 to τ in the present increment.

Alternatively, if $\tau_E^{t+\Delta t} < \tau(\gamma^t)$, yielding has not occurred so the inelastic strain remains constant and $\tau_E^{t+\Delta t}$ is retained as the final stress state. Once damage has initiated, unloading and reloading occurs along the damaged secant modulus from the fixed inelastic strain value as shown in path 3 in Fig. 11.

INSERT Figure 11: Shear stress-strain response with damage showing load reversal (1) and reloading (2) with kinematic hardening in the undamaged regime and reloading (3) in the damaged regime.

This shear model is based on the curve-fit of experimentally observed nonlinearity in the shear response. The unloading and loading reversal behaviour approximates the observed response.

2.2.3.2 Damage evolution

According to Puck and Schurmann's treatment of transverse damage, the normal and shear stresses on the fracture plane contribute to matrix cracking in a unidirectional ply [27]. As a consequence, the matrix damage parameter, d_{mat} , which controls the stiffness reduction due to matrix-dominated damage, is a function of both normal and shear stresses on the fracture plane. A resultant shear strain (γ_r) is defined on the fracture plane [26] as the vector sum of the two planar shear components shown in Fig. 7:

$$\gamma_r = \sqrt{\gamma_{NT}^2 + \gamma_{1N}^2} \quad (29)$$

The resultant failure strain, γ_r^{max} , is subsequently determined using the mixed mode critical energy release rate, G_{mat} , and the total strain energy before damage initiation, Λ ,

$$\gamma_r^{max} = \frac{2}{\sigma_r^0} \left(\frac{G_{mat}}{l_{mat}} - \Lambda \right) + \gamma_r^0 \quad (30)$$

where σ_r^0 and γ_r^0 are the resultant damage initiation stress and strain respectively. The volumetric strain energies associated with each stress component on the fracture plane, are combined using a quadratic relationship,

$$\Lambda = \Lambda_{NN} \left(\frac{\langle \sigma_{NN} \rangle}{\sigma_r} \right)^2 + \Lambda_{1N} \left(\frac{\tau_{1N}}{\sigma_r} \right)^2 + \Lambda_{NT} \left(\frac{\tau_{NT}}{\sigma_r} \right)^2 \quad (31)$$

where the resultant shear stress (σ_r) is defined as the magnitude of the stresses on the fracture plane:

$$\sigma_r = \sqrt{\langle \sigma_{NN} \rangle^2 + \sigma_{1N}^2 + \sigma_{NT}^2} \quad (32)$$

The volumetric strain energy associated with each stress component, Λ_{ij} , is given by the integral in Eq. (33).

$$\Lambda_{ij} = \int_{d_{mat}=0} \sigma_{ij} d\varepsilon_{ij} \quad , \quad \text{where } ij = NN, 1N, NT \quad (33)$$

The total strain energy release rate, G_{mat} , is:

$$G_{mat} = G_{mat}^C \left(\frac{\langle \sigma_{NN} \rangle}{\sigma_r} \right)^2 + G_{mat}^{12} \left(\frac{\sigma_{LN}}{\sigma_r} \right)^2 + G_{mat}^{23} \left(\frac{\sigma_{NT}}{\sigma_r} \right)^2 \quad (34)$$

and the matrix-dominated damage parameter, d_{mat} is therefore:

$$d_{mat} = \frac{\gamma_r^{max}}{\gamma_r^{max} - \gamma_r^0} \left(1 - \frac{\gamma_r^0}{\gamma_r} \right) \quad (35)$$

The shear stresses on the fracture plane are subsequently modified by the matrix-dominated damage parameter (d_{mat}).

$$\sigma_{LN} = (1 - d_{mat}) \bar{\sigma}_{LN} \quad (36)$$

$$\sigma_{NT} = (1 - d_{mat}) \bar{\sigma}_{NT} \quad (37)$$

$$\sigma_{NN} = \bar{\sigma}_{NN} - d_{mat} \langle \bar{\sigma}_{NN} \rangle \quad (38)$$

These stresses are transformed back to the material coordinate system to form the complete stress tensor of the damaged element. The shear (Eqs. (36) and (37)) and normal (Eq. (38)) degradation on the fracture plane results from the combined action of transverse and shear stress states.

2.2.3.3 Implementation of matrix-dominated damage

An optimisation based on Brent's algorithm [45] was used for maximising the damage initiation function to obtain the fracture plane angle. This method, which combines the robustness of a golden section search with the speed of quadratic interpolation, is superior to a series of function evaluations on possible fracture plane angles [45] in balancing a fast run time with good accuracy. To reduce unnecessary evaluations of the damage initiation function, a bounding box was introduced to quickly check whether a particular loading state was well below that needed to achieve damage

initiation. This bounding box was created to encompass the set of all possible stress states in the 1NT coordinate system due to rotation of the fracture plane. Fig. 12 shows the blue curve, representing the possible stresses at different fracture plane angles contained within a box in the 1N-NN, NT-NN and 1N-NT-NN stress spaces respectively. The red curve/surface represents damage initiation curve/surface in the respective stress spaces, which the bounding box is compared against.

INSERT Figure 12: Bounding box over the set of stress states possible due to rotation of the fracture plane as shown in the (a) 1N-NN (b) NT-NN and (c) 1N-NT-NN stress spaces.

If the bounding box is entirely within the damage initiation surface, then this stress state cannot initiate damage. Hence the routine, which maximises damage initiation functions to identify the inclination of the fracture plane, is not executed.

2.3 Implementing the damage model within Abaqus

The intralaminar damage model was implemented using the user-defined material subroutine (VUMAT) within the Abaqus/Explicit package [39]. The Abaqus core provides the VUMAT with the current increment strain values as well as all state variable values from the previous increment. VUMAT then calculates and returns the stress state of the current increment to Abaqus at each integration point.

The proposed model assumed a homogenised composite ply where microcracking is assumed to be smeared over the volume of the element. The lack of discontinuities within the element allows the application of conventional FE analysis rather than necessitating more exotic methods such as the extended finite element method [46].

Any damage that occurs is assumed to be irreversible according to thermodynamic principles. Hence the damage parameters are constrained to be monotonically increasing. As this model was developed to model composite crushing, fatigue need not be considered. Strain rate dependence has been neglected as the fibre-dominated properties, which is the principal mode of energy dissipation due to its substantially higher critical strain energy release rate, was shown to be rate independent [47].

Element deletion was employed to remove elements based on: (i) the damage parameter; or (ii) the determinant of the deformation gradient, $\det(F)$.

$$\text{delete element when either } \begin{cases} d_{11} > 0.99 \\ 0.8 > \det(F) \text{ or } \det(F) > 1.6 \end{cases} \quad (39)$$

These parameters indicate: (i) the lack of resistance to loading leading to excessive distortions and (ii) the occurrence of large volume changes respectively. Elements displaying these characteristics were deleted, as their response was no longer valid and could cause the simulation to abort.

For composite structures undergoing crush damage, delamination, friction and contact are also important considerations and established algorithms built into Abaqus were used to capture these effects [39]. A bilinear traction-separation law was used to capture the interlaminar behaviour and applied to cohesive surfaces. The maximum traction was determined by the delamination strength, after which softening occurs. Delamination occurs when the strain energy in the surface exceeds the critical fracture energy. Mode mixing was achieved through a power law [48]. A general contact algorithm built into Abaqus was utilised to generate the required tangential and

normal forces between contact surfaces [39]. Normal contact forces imposed hard contact conditions between the platen and the plies as well as between adjacent plies for when the plies come into contact after the cohesive surfaces were “eroded” to prevent penetration. Tangential contact forces consist of friction forces experienced when the ply-platen or ply-ply interface slide over each other, which was determined using the Coulomb friction model [39].

3 Material property measurement

The present material model requires the input of a number of material properties which impact the accuracy of the prediction. Some of the commonly-used properties have associated standards for their measurement, e.g. ASTM D3039M [49] for tensile strength, modulus and Poisson’s ratio, ASTM D3410M [50] for compressive strength and modulus, and ASTM D3518M [51] for the shear profile and strength measurement of a composite ply. However, methods describing the measurement of intralaminar critical energy release rates are not as well established. Pinho et al [52] demonstrated the use of compact specimens, described in ASTM E399 [53] and E1820 [54], for the determination of both tensile and compressive fibre-dominated critical energy release rates. It was shown that the method described in ASTM E399, originally intended for determining the fracture toughness of metallic materials, was not suitable due to the anisotropy of composite laminates. An alternative FE based method was proposed and validated. This approach is also similar to the single edge notch test method described in ASTM E1922 [55], which applies only for tensile properties. ASTM E399 can also be applied to the matrix-dominated energy release rates in tension and compression by rotating the orientation of the laminate. There are

currently no direct methods to measure the intralaminar shear energy release rates. However, the mode II interlaminar energy release rate is an appropriate approximation due to the similarities in the failure mode. In contrast, methods to obtain mode I, mode II and mixed-mode interlaminar critical energy release rates for unidirectional composite laminate are specified in ASTM standards D5528 [56], D7905 [57] and D6617 [58] respectively.

3.1 Material property sensitivity

Due to the epistemic uncertainty in some of the input material properties, the sensitivity of the value of transverse strengths was investigated using a model of a simple chamfered flat plate undergoing crushing against a rigid surface. The transverse material properties were chosen because of their probable in-situ dependence [44].

INSERT Figure 13: Sensitivity of model to transverse properties.

Fig. 13 shows the variation in peak force and SEA with variations to the baseline transverse properties. As expected, decreasing the transverse tensile, compressive and shear strengths to 20% of the baseline values resulted in a reduced peak force. On the other hand, a fivefold increase in the transverse tensile and shear strengths led to failure via buckling away from the crush front, resulting in a decreased overall peak force. The increase in transverse compressive strength caused an expected increase in peak force. Overall, the simulation demonstrated that the SEA was not very sensitive to the variation in these transverse properties, which is the result of the low energy

associated with matrix damage. The primary energy dissipation is expected to be through fibre-dominated damage mechanisms.

4 Model validation

4.1 Mesh sensitivity

The developed constitutive model softens the material locally as damage progresses. This leads to mesh-dependent localisation of damage, which is resolved through the use of a characteristic length. A mesh sensitivity study, on a cube loaded in longitudinal tension was performed with 1^3 , 2^3 , 3^3 , 4^3 and 5^3 elements respectively. The response in Fig. 14 confirms the mesh independence of the proposed model.

INSERT Figure 14: Force-displacement curve for cube model with different mesh densities.

The small deviation near complete failure is attributed to the breakdown of the infinitesimal strain assumption used in the model due to the large strains experienced by elements nearing complete failure.

4.2 Uniaxial transverse compression coupon simulation

The matrix-dominated damage mode for a unidirectional laminate, loaded in transverse compression, was validated against experimental data. A numerical model of the compression coupon was investigated to demonstrate that the intralaminar damage model was able to capture the observed fracture plane observed experimentally [59] and described in the literature [27].

4.2.1 Model setup

A virtual coupon model, similar to that described in ASTM D3410 [50], was created with dimensions 20x10x4 mm. This virtual coupon was meshed with uniform cubic C3D8R linear reduced integration solid elements with a side length of 0.2mm. The virtual coupon was assigned with properties of T700/M21 obtained from the literature [60-64] and in-house testing. Some matrix properties were unavailable and were substituted with those available for a similar epoxy resin [26]. The elastic moduli were $E_{11} = 142$, $E_{22} = E_{33} = 8.4 GPa$, $G_{12} = G_{13} = 4.8 GPa$ and $G_{23} = 2.9 GPa$ with a Poisson ratio of 0.32 [61]. The longitudinal strengths were 2282 MPa and 1465 MPa [61], while transverse strengths were 65 MPa [61] and 290 MPa [26], for tension and compression respectively. The transverse shear strength was 105 MPa [61]. In house testing yielded $G_{fib}^T = 108$ and $G_{fib}^C = 58.4 kJ/m^2$ for the fibre. $G_{mat}^T = 0.331$ and $G_{mat}^{ij} = 0.443 kJ/m^2$ [63] were used for the matrix. G_{mat}^C was estimated to be 1.1 kJ/m² [26]. Cubic shear coefficients $c_1^{ij} = 34.24 GPa$, $c_2^{ij} = 15.06 GPa$ and $c_3^{ij} = 2.198 GPa$ were determined from the shear response curve [62]. The density was set to 1.59 g/cc [26]. The specimen was loaded in uniaxial transverse (the y direction in Fig. 15) compression.

4.2.2 Results

INSERT Figure 15: Compressive failure along the fracture plane [59] (left) is well captured by the virtual coupon (right) with fibres parallel to the x direction.

Fig. 15 (left) shows a fractured uniaxial compression coupon of a similar material (IM7/8552). The fracture morphology of the numerical model shown in Fig. 15 (right)

bears close resemblance to experimental observation. Formation of multiple fracture surfaces observed experimentally was also captured by the virtual coupon. The inclination of the macroscopic fracture plane was consistent with an expected value of approximately 53° [27].

4.3 Composite crush specimen simulation

The crushing of a tulip triggered cylindrical energy absorber specimen was simulated to validate the proposed model for use in evaluating energy absorber response.

4.3.1 Experimental setup

A series of quasi-static crush tests were completed on a set of composite energy absorber specimens. A tulip triggered cylinder (Fig. 16) geometry was chosen following recommendations from the literature [6]. The cylindrical tubes were manufactured from T700/M21 unidirectional prepreg with a $[0/90/0/90]_s$ layup to obtain a nominal wall thickness of 1.2mm. The manual layup process includes a debulking process to minimise imperfections within the plies. The specimens were cured in an autoclave as per manufacturer's instructions. The tulip trigger pattern was cut into the top of the tubes after curing. Care was taken to ensure that the top and bottom of the tube were machined parallel for even load distribution. This set of specimens was crushed between two steel platens at quasi-static speeds of 0.5 mm/min in a screw driven testing machine.

INSERT Figure 16: Test specimen geometry.

4.3.2 Virtual specimen setup

A virtual tulip triggered test specimen was created for Abaqus/Explicit. The model was meshed with an approximate element dimension of 1mm in the longitudinal and transverse directions to balance accuracy with runtime. Each ply had three elements in the thickness direction to adequately capture post-delamination ply bending. The plies were modelled using the same element and properties as the uniaxial compression virtual coupon. The eight ply layers were modelled individually. The platens were modelled as rigid flat surfaces. Interlaminar behaviour between adjacent plies were captured via cohesive surfaces [39]. The interface strength was set to 60 *MPa* for both modes I and II whereas the energy release rates were 0.331 and 0.443 *kJ/m²* for modes I and II respectively [63]. Hard contact conditions [39] were defined between the platen and the plies as well as between adjacent plies for when the plies come into contact after the cohesive surfaces were “eroded”.

Friction played a significant role in the response of the structure. Numerical analysis shows that the friction coefficient between the composite plies and the platen is not constant. During the consumption of the trigger region (<10 mm displacement), substantial friction was present. An experimentally measured value of 0.24 was adopted for this region. However, during steady-state crushing of the bulk cylinder, a significantly lower friction coefficient yielded a good match to experimental observations. A friction coefficient of 0.10 was used during steady state crushing. This reduced friction coefficient can attributed to the lubricating effects of trapped graphite debris/dust on the composite-ply interface. This is supported by the observed lubricity of small graphite particles [65] which are similar in composition to the fine carbon fibre dust observed during testing.

The model utilised quarter symmetry to reduce the computational resources required. To suppress spurious responses, an enhanced stiffness based hourglass control and distortion control was employed [39]. Variable mass scaling on a per-element basis, similar to that implemented by other authors ([2] and [66]), was employed to further speed up the simulation time. A sensitivity study was used to ensure the effect of mass scaling on the final response was small. The simulation was run using Abaqus/Explicit 6.11. Low-pass filters were necessary to remove the numerical oscillations, which are an artefact of explicit dynamic modelling.

4.3.3 Results

The force and displacement histories of the experimental tests were recorded via the attached load cell and the frame respectively. The dominant damage modes were also noted. The experimental results showed good consistency in terms of force-displacement as well as the observed damage.

Fig. 17 shows the evolution of damage in the specimens under monotonically increasing crushing loads and Fig. 18 shows the simulated force-displacement response, which is consistent with the range of observed experimental results. The displacement at which the peak force occurred was predicted with good accuracy. The higher predicted peak force is likely a numerical artefact relating to the changing contact conditions upon element deletion. The progressive nature of the crushing was well captured, with a clear force plateau during the steady state crushing.

INSERT Figure 17: Comparison of experimental and simulated deformation of the specimen: (i) splitting of plies, (ii) petalling of trigger section and (iii) substantial matrix damage in splayed plies (red region).

INSERT Figure 18: Overlay of simulated crush response (dark) over experimental data (light).

Two primary performance metrics, the energy absorption and average force, were well predicted by the model (Fig. 19).

INSERT Figure 19: Energy absorption (left) and force comparison (right) between numerical (dark) and experimental results (light).

5 Conclusion

An intralaminar damage model was developed and combined with established interlaminar and contact models to form a complete modelling package able to predict the crush response of composite structures. The present model is fully three-dimensional, combining an improved characteristic length determination, nonlinear shear, a robust unloading/reloading mechanism and a unified matrix damage mechanism, which provides greater fidelity and predictability than previously reported. The model successfully reproduced the experimental response of a set of tulip triggered tubular composite energy absorber specimens. This was achieved without the need to alter or calibrate experimentally determined input parameters like many currently available damage models, which gives the present model a predictive capability. The use of this numerical model can contribute to the reduction in the amount of physical testing necessary in the design of energy absorbing composite structures, which has potential for significant improvements to the time and cost of the design process.

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FIGURES

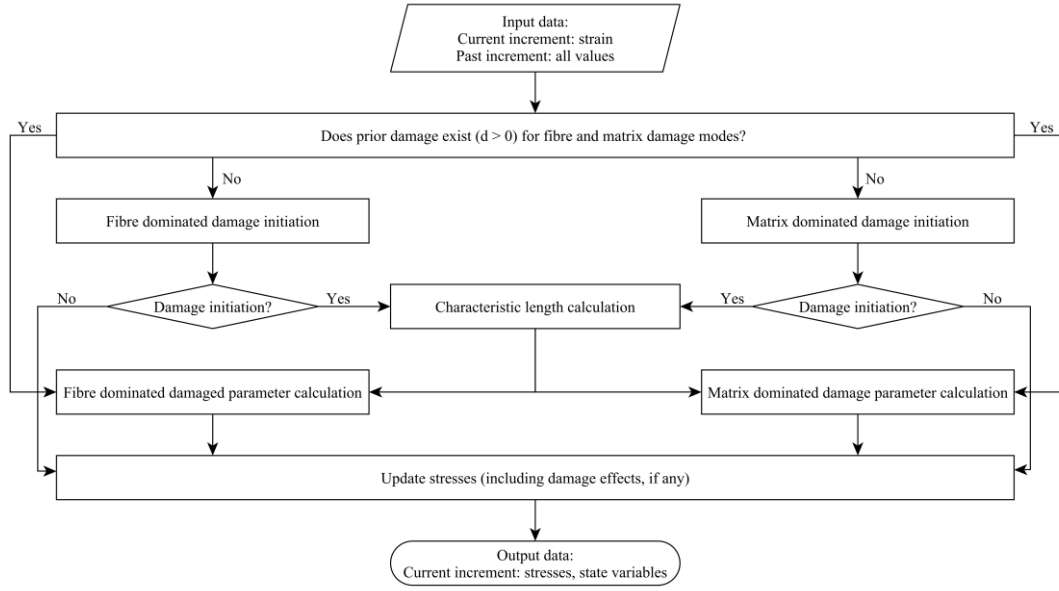


Figure 1: Flowchart showing the processes within the intralaminar damage model

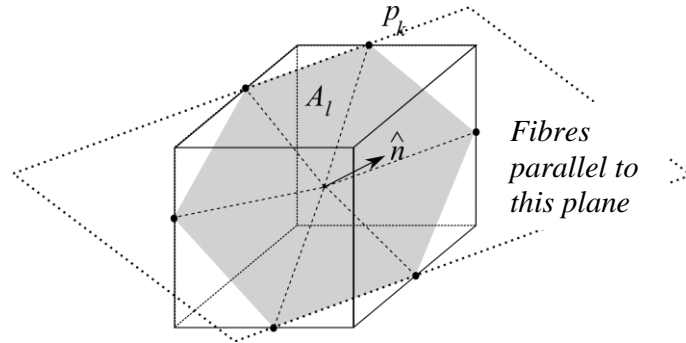


Figure 2: Definition of unit normal vector (\hat{n}) and points of edge intersection (p_k) for an arbitrary fracture plane (shaded) within a hexahedral element

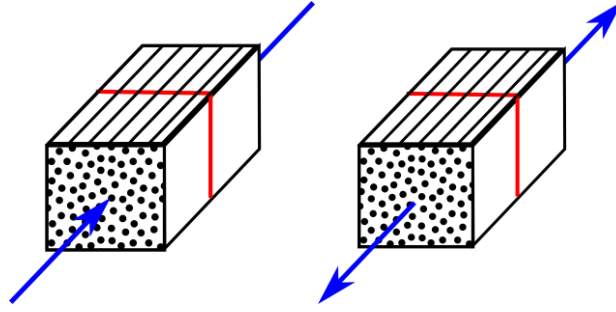


Figure 3: Fibre-dominated fracture with associated fracture plane

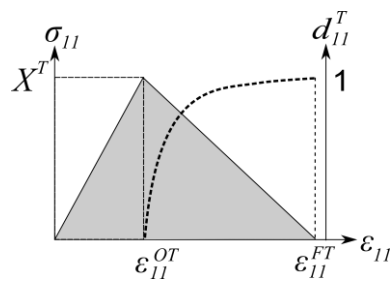


Figure 4: Bilinear stress-strain law (shaded area is the critical volumetric strain energy release rate

g_{fib}^T) and associated damage parameter growth (bold dashed line)

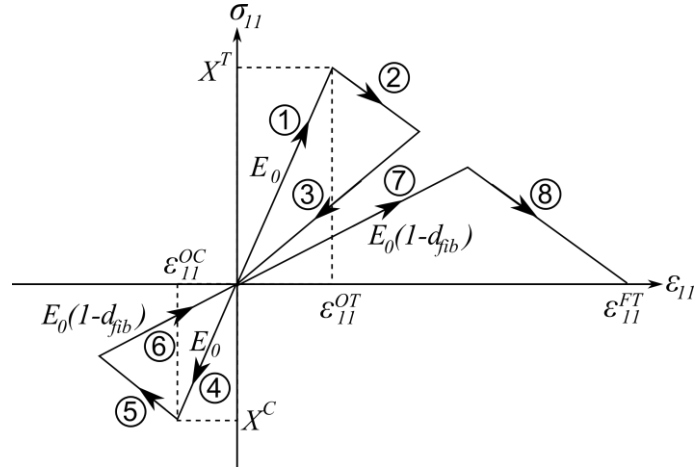


Figure 5: Stress-strain response during fibre direction loading/unloading: paths 1-2, tensile loading; 3, unloading; 4-5, loading in compression; 6-8, compressive unloading and tensile reloading until failure

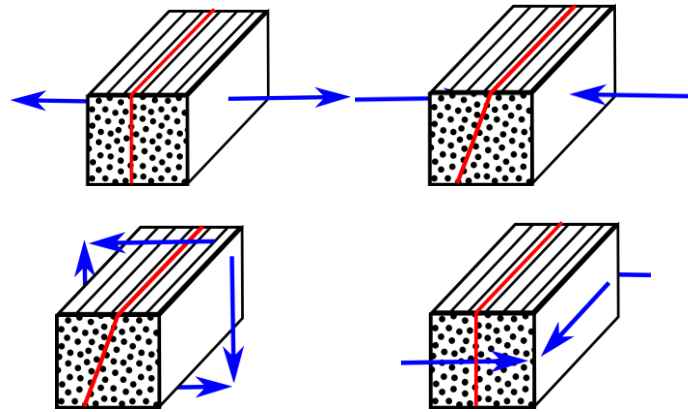


Figure 6: Matrix-dominated fracture with associated fracture plane

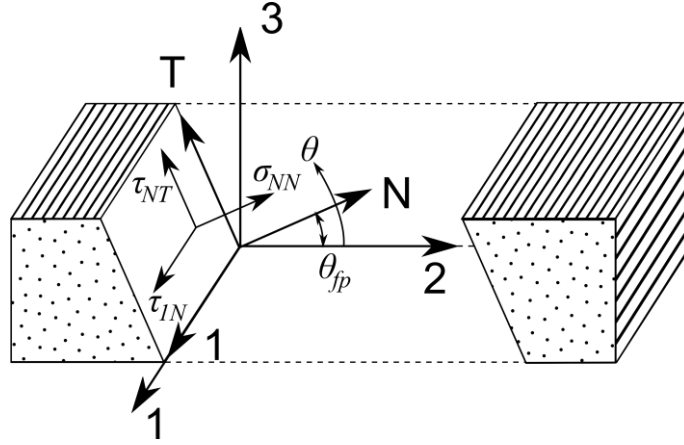


Figure 7: Coordinate system attached to the fracture plane (1,N,T) [12] relative to the material coordinate system (1,2,3)

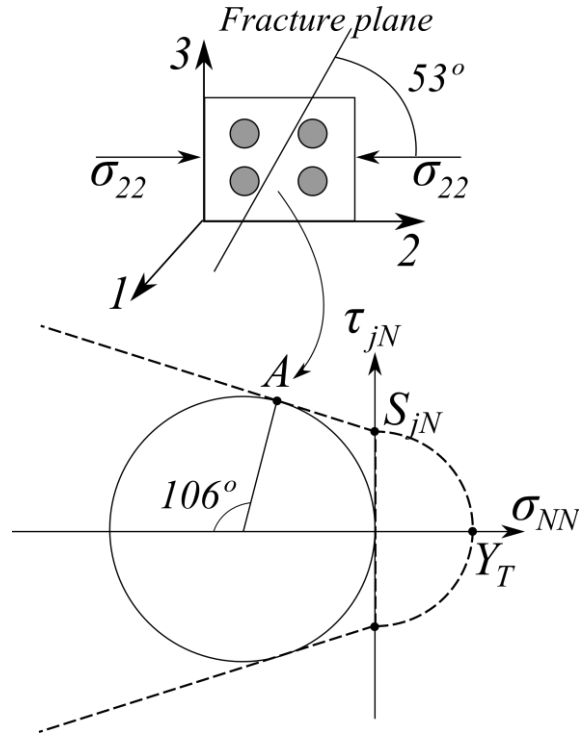


Figure 8: Defining the damage initiation profile (dashed line) via the material properties on the friction plane, where $j = 1, T$, and inset represents the stress state at point A

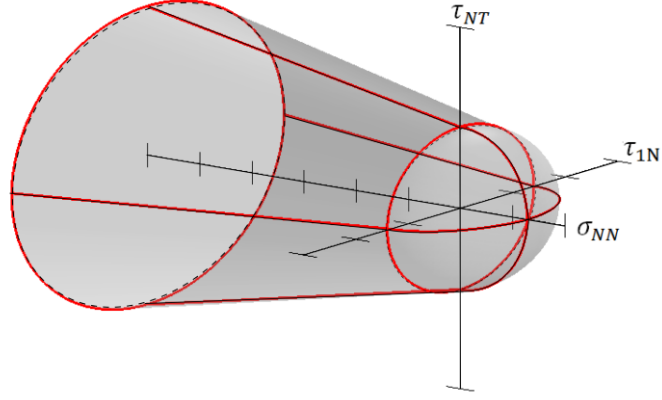


Figure 9: Damage initiation surface in the stress space defined by the friction plane

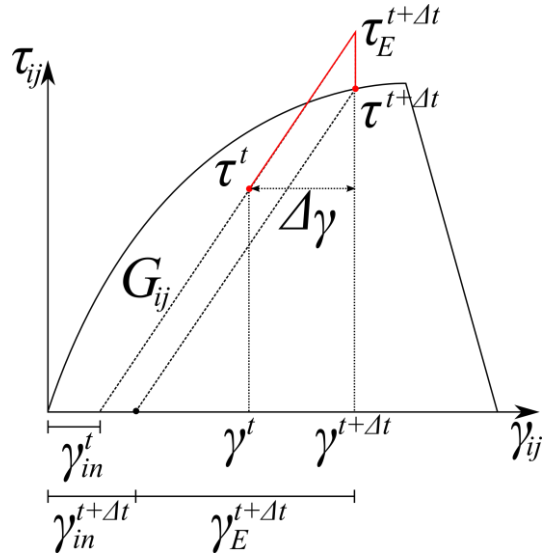


Figure 10: Calculating shear stress from shear strain using the elastic predictor method progressing from old stress state τ_0 to τ in the present increment

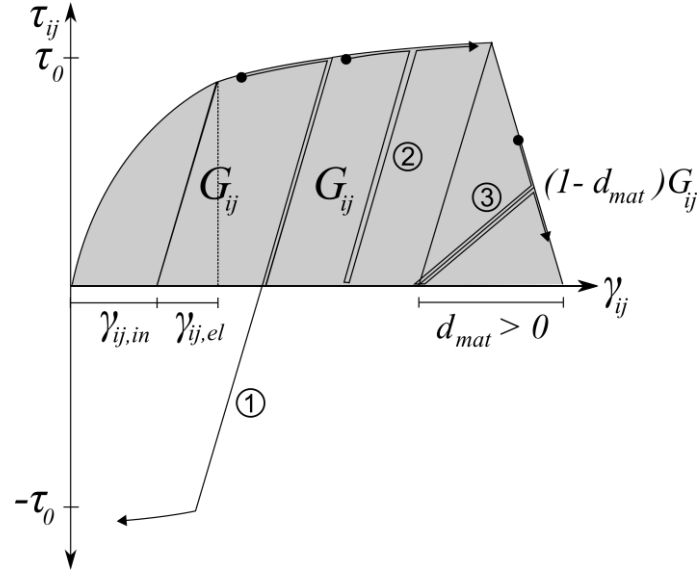


Figure 11: Shear stress-strain response with damage showing load reversal (1) and reloading (2) with kinematic hardening in the undamaged regime and reloading (3) in the damaged regime

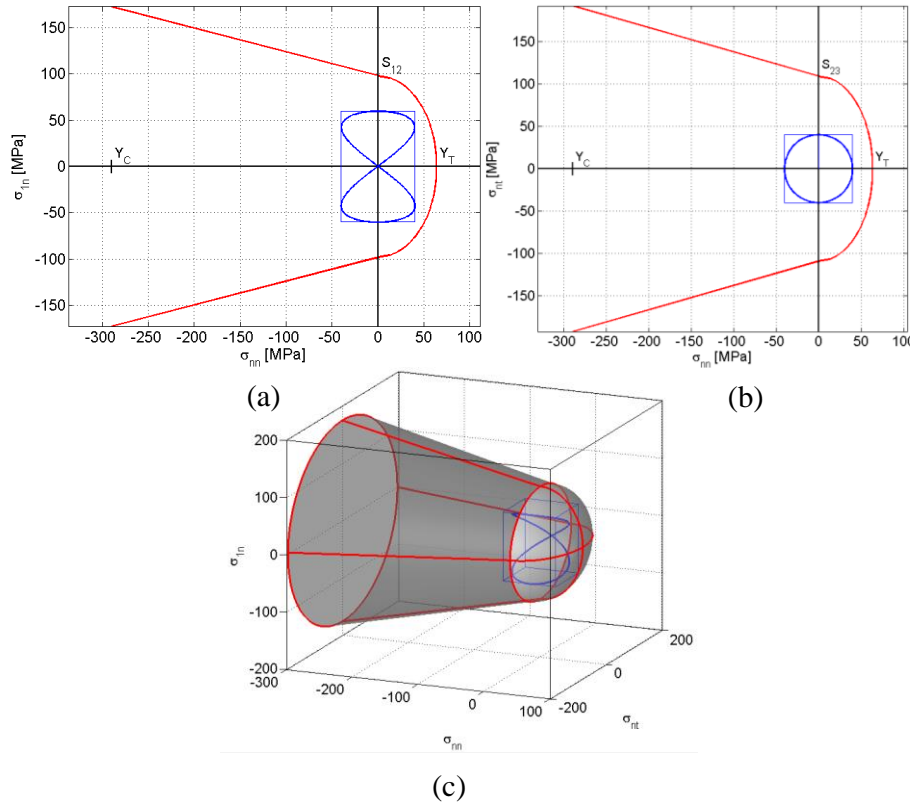


Figure 12: Bounding box over the set of stress states possible due to rotation of the fracture plane as

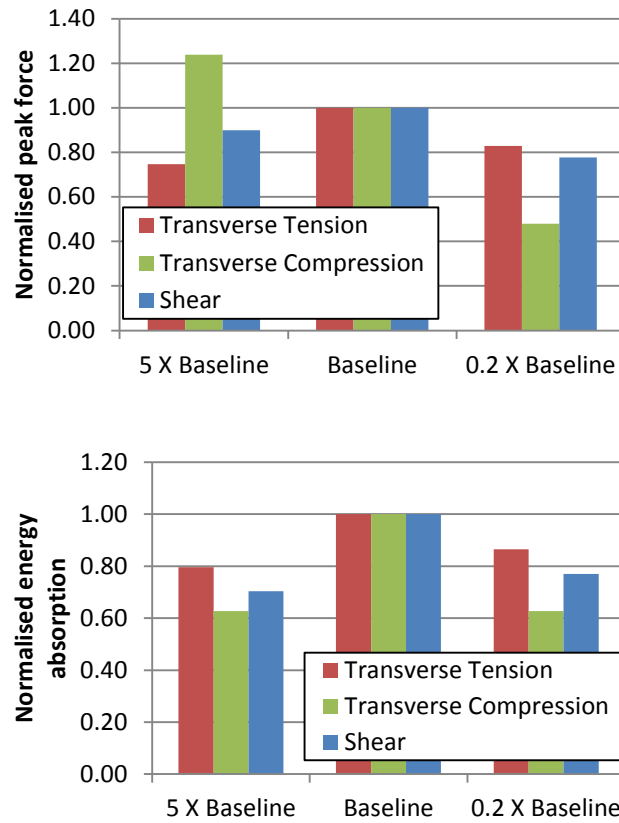


Figure 13: Sensitivity of model to transverse properties

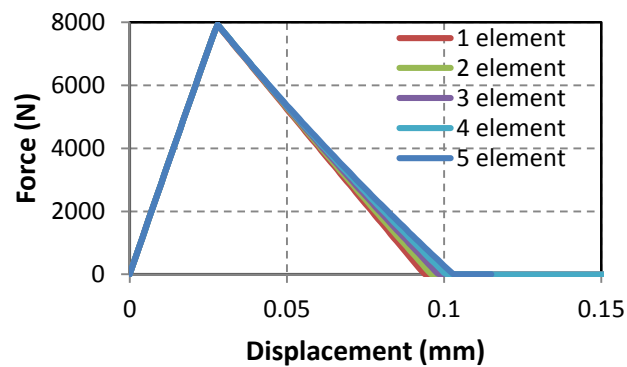


Figure 14: Force-displacement curve for cube model with different mesh densities

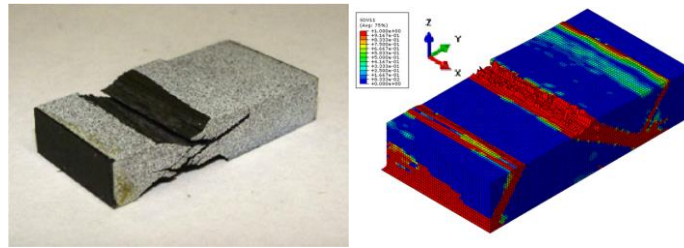


Figure 15: Compressive failure along the fracture plane [59] (left) is well captured by the virtual coupon (right) with fibres parallel to the x direction

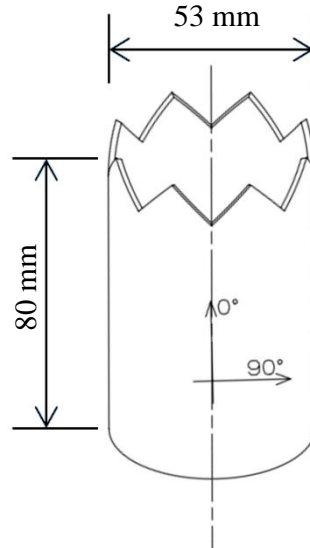


Figure 16: Test specimen geometry

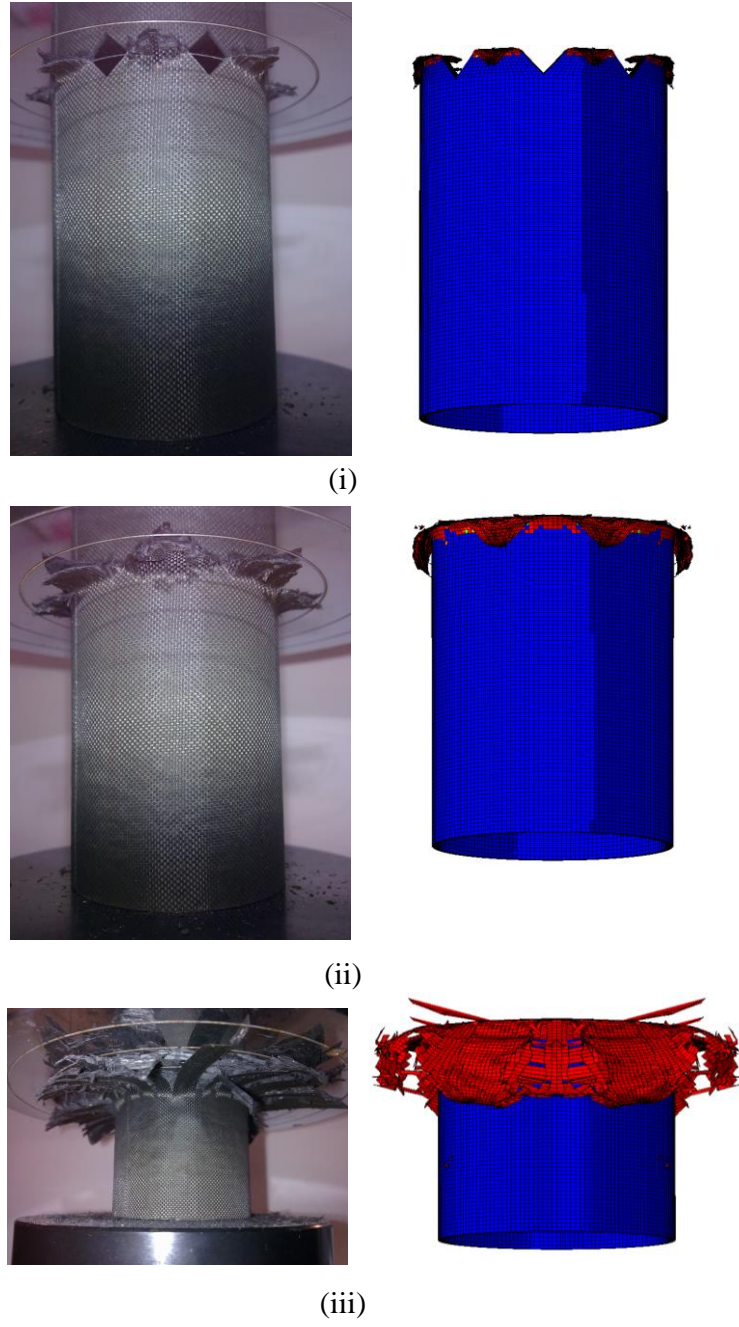


Figure 17: Comparison of experimental and simulated deformation of the specimen: (i) splitting of plies, (ii) petalling of trigger section and (iii) substantial matrix damage in splayed plies (red region)

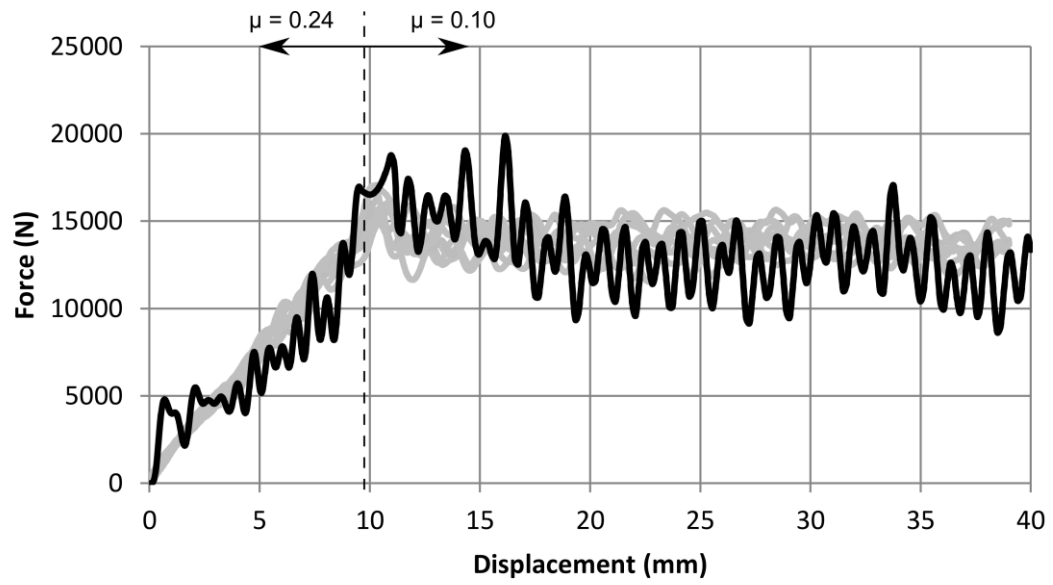


Figure 18: Overlay of simulated crush response (dark) over experimental data (light)

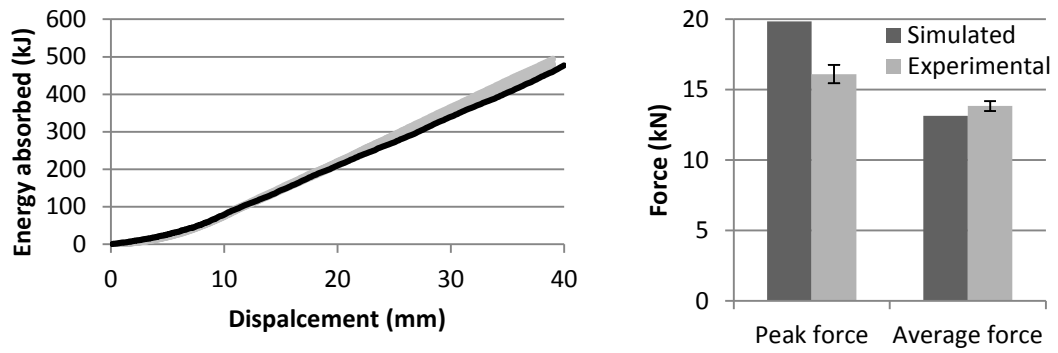


Figure 19: Energy absorption (left) and force comparison (right) between numerical (dark) and experimental results (light)

TABLES

Table 1: Summary of comparison between existing models and the present model

Model	Element formulation	Robust characteristic length*	Nonlinear shear	Load reversal	Damage interaction	
					Fibre	Matrix
Commercially available models						
PAM-CRASH ply type 7 [24]	2D	N	N	N	N	N
LS-DYNA Mat 54 [20]	2D	N	N	N	N	N
Abaqus Hashin Damage [39]	2D	N	N	N	Y	Y
Recently published models						
Donadon et al. 2008 [25]	3D	N	Y	Unloading/reloading only	Y	Y
Faggiani and Falzon [26]	3D	N	Y	Unloading/reloading only	N	Y
Shi et al. [28]	3D	N	Y	N	Y	Y
Raimondo et al. [29]	3D	N	Y	N	Y	Y
Present model	3D	Y	Y	Y	Y	Y

* Capable of accurately assessing characteristic length for a fracture plane in an arbitrary orientation with respect to both the material and the elemental coordinate system